Dye Injection at the Centerline of a Pipe

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Turbulent mixing occurs when blobs of a scalar quantity (such as heat or mass) are reduced to the molecular scale by the process of turbulent eddy motion and molecular diffusion. The turbulent flow field contains eddies that behave in a random nature. These eddies strongly affect the mixing, both on a large molar scale and on the molecular scale. Eddies convey the scalar quantity from one part of the system to another, which results in an increased ability to break down the blobs of the scalar quantity. The subsequent molecular mixing is enhanced by the increase in surface area and the steep scalar quantity gradient on which the rate of molecular diffusion depends.

Our main references for this work are the previous paper by Lee and Brodkey (*1*), the note by Brodkey (*2*), and the thesis by Lee (*3*). In these, the previous literature has been reviewed and will not be repeated here. Certain specific theoretical and experimental results will be cited as necessary. It is the purpose of this paper to retreat some of Lee's data, to treat other data of his in detail for the first time, and to present the results for comparison to theory and the work of others.

As a preliminary to the mixing analysis in terms of the intensity of segregation, the mean and fluctuating concentration measurements made by Lee and Brodkey (*1*, *3*) were studied. Examples of the distribution curves, conditions of experimentation, and methods of measurement can be found in the references cited. The analysis of the mean concentration information parallels that made by others and similar results were obtained. Details of these can be found in reference *4*. Briefly, these results were: (1) The bell-shaped mean concentration distributions at various axial positions were normalized and were represented by a normal distribution equation in which the radius was made nondimensional with the radial position where the mean concentration was one-half the maximum. (2) The mean concentration decay along the centerline could be represented by a three-constant hyperbolic equation. (3) The concentration fluctuation to local mean concentration ratio distribution showed a limited self-preservation region away from the injector. For longer distances, the ratio fell consistently away from the self-preservation line, which was to be expected as the fluctuation tends to zero at very long distances. (4) The rate of spread of the dye plume was used with isotropic theory to provide a crude estimate of the parameters of turbulent diffusion. Because of the injector disturbance, short time diffusion estimates were very crude; however, the estimates for the radial velocity fluctuation bracketed the reported results of others who obtained data in pipe systems. (5) Two different methods of analysis of long diffusion time data were made. The turbulent Peclet numbers obtained were consistant with the range of values reported in the literature. Again, details of these conventional analyses can be found in reference *4*.

Of prime importance here is the modification of the scalar mixing analysis to apply at any radial position rather than at just the centerline as previously used (*2*). The detailed development of the theory will not be given here, since it can be readily found in references *1*, *2*, and *5*. Briefly, the theory suggests that the intensity of segregation is given by

$$I_s = c'^2/c_o'^2 = e^{-t/\tau} \tag{1}$$

where the time constant τ is

$$\tau = \frac{1}{2}\left[3\left(\frac{5}{\pi}\right)^{2/3}\left(\frac{L_s^2}{\epsilon}\right)^{1/3} + \left(\frac{\nu}{\epsilon}\right)^{1/2} \ln N_{Sc}\right] \tag{2}$$

Unfortunately L_s, the macroscale of mixing, has never been measured, so that an estimate for this was given in reference *2* as

$$\left(\frac{5}{\pi}\right)^{2/3}\left(\frac{L_s^2}{\epsilon}\right)^{1/3} = \left(\frac{\lambda^2}{10\nu}\right) = \frac{0.341\, r_o}{u'} \tag{3}$$

In order to complete the analysis an estimate for ϵ for the second term of Equation (2) is needed. The microscale λ can be related to ϵ and u' by

$$\lambda^2 = 15\,\nu\, u'^2/\epsilon \tag{4}$$

which, when combined with Equation (3), gives

$$\epsilon = (1.5/0.341)\, u'^3/r_o \tag{5}$$

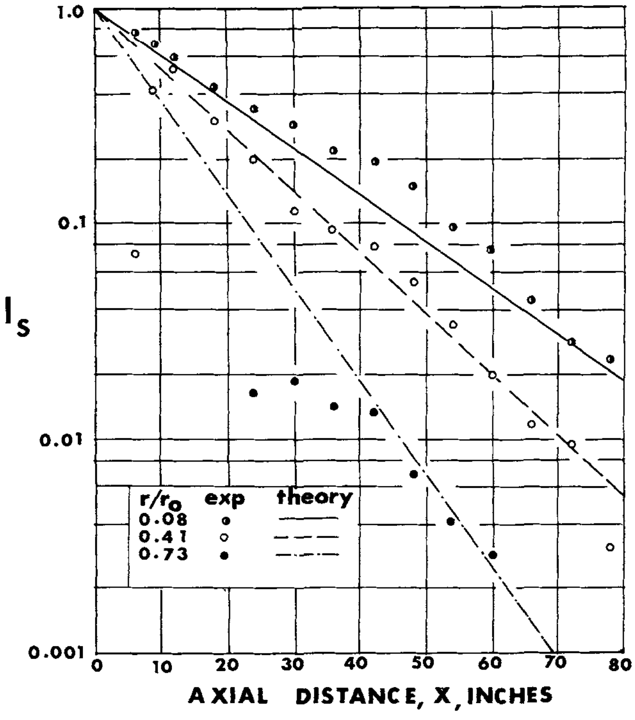

Fig. 1. Decay of the intensity of segregation for selected radial positions.

Since the entire radial profile of root-mean-square velocities for Lee's system is available (*6*), a relatively complete analytical treatment of the data is possible. Time constants can be calculated for any radial position for which u' data are on hand, simply by applying Equations (3) and (5) to (2). For example, at a dimensionless radius of 0.326, u' is 2.806 cm./sec. Upon substitu-

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tion of the pipe radius, 3.896 cm., and u' into Equations (3) and (5), we obtain ϵ = 24.9 sq.cm./sec.^{3} and $\lambda^2/10\nu$ = 0.474 sec. The Schmidt number for the system is 7,760 and the kinematic viscosity is 0.01 sq.cm./sec. Substitution of these values into Equation (2) results in a time constant of 0.800 sec. for the radial position chosen.

In order to compare the experimental data for every radial position, the intensity of segregation was calculated from the data in exactly the same manner as suggested by Lee and Brodkey (*1*). Three such examples are shown in Figure 1; more can be found in reference *4*. The theoretical lines that appear in the figure were calculated by changing the time dependency of Equation (1) to a distance dependency (*2*). The actual elapsed time in Equation (1) can be replaced by the length traveled by a blob of fluid moving at the local mean velocity $\bar{U}$; that is

$$t = x/\bar{U} \tag{6}$$

The decay of intensity of segregation expressed by Equation (1) is changed to a function of axial distance by multiplying the time constant by $\bar{U}$.

DISCUSSION

One of the results of the least squares treatment of the raw intensity of segregation data was the slope of the best line through each set of points. These slopes can be compared to the distance decay constants calculated from theory. The comparison appears in Table 1.

The largest difference, 23.8%, occurs at the radii closest to the wall. The experimental data are seen to lie above the theoretical lines for r/r_o out to 0.163 and then to fall below for all but the last two radii near the wall. The percent difference is no greater than ± 10% out to three-fourths of the way to the wall. The data for r/r_o between 0.651 and 0.895 show much scatter, particularly in the first few points of each set. This makes it difficult to establish a good straight line; hence, slopes are expected to be inaccurate in that range. A further indication of the difficulty in obtaining slopes there can be seen from the large jump between the percent differences for r/r_o = 0.732 and r/r_o = 0.814. The jump is from +8.1% at the former radius to −21.4% at the latter.

All of the I_s results can be represented on a single line if the dimensionless group $x/\bar{U}\tau$ is chosen as the abscissa instead of x. The results fall very closely about a single line of the theoretical slope of −1 as shown in Figure 2. The few points that deviate from the line are those near the wall.

The remarkably good fit between the data and Corrsin's theory is encouraging. Pipe flow is markedly nonisotropic except for the centerline region; the rest of the pipe is in shear flow. Corrsin's analysis is not an exact solution to the isotropic mixing problem and is even more approximate for the shear flow. The fact that the fit is within 10% for three-fourths of the flow field indicates that even with its many assumptions, Corrsin's theory gives a very good estimate of mixing. This constitutes a use of isotropic theory, not a proof. Such a good fit to the data could be misleading, and thus several further comments are in order: (1) In our experiments, the velocity field did not match that of the theory, since no − 5/3 spectral region was obtained. (2) Likewise, the concentration field did not match for the same reason. (3) The reasonable estimates obtained can be attributed to the fact that the theory requires an integration of the spectrum to obtain the concentration fluctuations. Such an integration is a smoothing process such that information about the spectrum shape is lost. (4) The decay curves do not follow an exact straight line, and the least squares fit is really an average value. In all cases the experimental value of $1/\bar{U}\tau$ near the injector was less than that in Table 1 and greater far from the injector. Very recent results obtained with a smaller probe located at the centerline gave similar results, although the curvature was more pronounced.

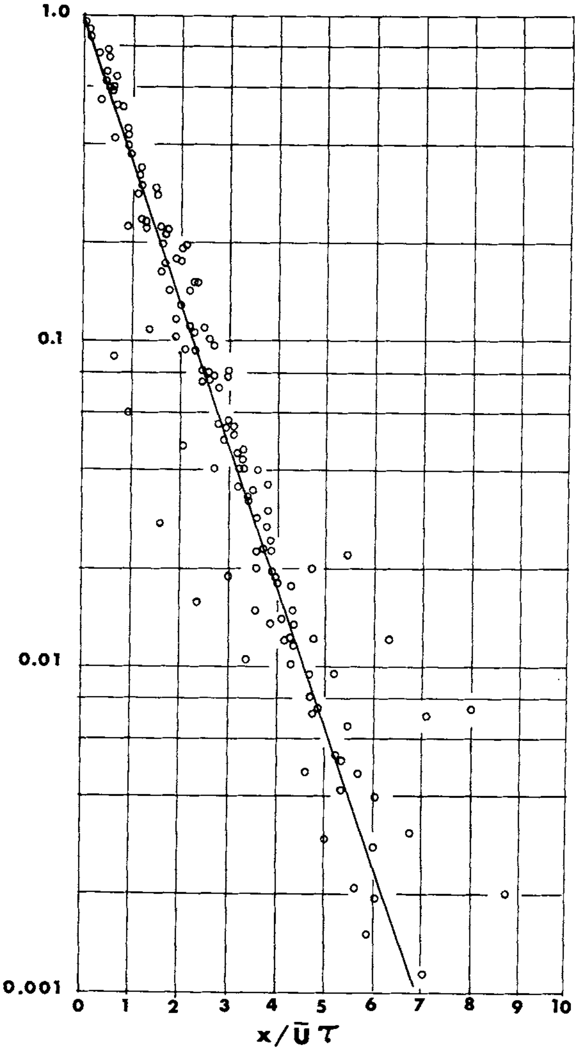

Fig. 2. Normalized decay of the intensity of segregation.

TABLE 1. COMPARISON OF INTENSITY OF SEGREGATION DECAY CONSTANTS

r/r_o	$1/\bar{U}\tau$ in.$^{-1}$ Theoretical	Experimental	% Difference
0	0.0490	0.0443	−9.6
0.0814	0.0500	0.0461	−7.8
0.163	0.0519	0.0501	−3.5
0.244	0.0548	0.0565	+3.1
0.326	0.0593	0.0638	+7.6
0.407	0.0648	0.0660	+1.9
0.488	0.0719	0.0726	+1.0
0.570	0.0791	0.0799	+1.0
0.651	0.0891	0.0928	+4.2
0.732	0.0995	0.1076	+8.1
0.814	0.1140	0.0896	−21.4
0.895	0.1321	0.1007	−23.8

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NOTATION

c' = root-mean-square concentration fluctuation
I_s = intensity of segregation
L_s = scale of mixing
N_{Sc} = Schmidt number (ν/D)
r = radius
r_o = pipe radius
t = time
$\bar{U}$ = local mean velocity
u' = root-mean-square axial Eulerian velocity fluctuation
x = axial distance
ϵ = turbulent energy dissipation
τ = time constant
ν = kinematic viscosity

LITERATURE CITED

1. Lee, Jon, and R. S. Brodkey, *A.I.Ch.E. J.*, **10**, 187 (1964).
2. Brodkey, R. S., *ibid.*, **12**, 403 (1966).
3. Lee, Jon, Ph.D. thesis, Ohio State Univ., Columbus (1962).
4. Gegner, J. P., M.S. thesis, Ohio State Univ., Columbus (1965).
5. Corrsin, Stanley, *A.I.Ch.E. J.*, 3, 329 (1957); **10**, 870 (1964).
6. Knox, J. S., M.S. thesis, Ohio State Univ., Columbus (1966).

Accuracy in Residence Time Measurements

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Residence time distributions in flow or reaction systems give information about internal flow characteristics and are useful for predicting conversions with first-order reactions or in completely segregated reactors. The mean residence time provides a way to measure system volume by external means and the variance has found wide utility in estimating dispersion parameters in system models (*1*).

It is, of course, not possible to make these measurements with absolute precision. In particular, every real system has internal life expectancies extending to infinity, so that analytical problems or impatience eventually truncates all measurements.

The usual technique is to inject a pulse of tracer material into the system and then to follow the effluent time-dependent concentration. Alternatively, tests in the form of saturation or elution steps may be used. If the system is linear these responses are theoretically interchangeable by well-known techniques. If $c(t)$ is the effluent concentration response to a true pulse input, the normalized residence time density distribution $f(t)$ is estimated, in practice, from

$$f^*(t) = \frac{c(t)}{\int_0^{t^*} c(t)\,dt} \qquad 0 \leq t < t^* \tag{1}$$

$$= 0 \qquad t \geq t^*$$

The time t^* is the last time at which the tracer may be detected or when, for analytical reasons, it appears that the response has gone to zero. The limit $t^* \to \infty$, if it were possible to obtain, gives the true distribution $f(t)$. As a check on the accuracy of the measurement, the material balance

$$\frac{q}{Q} \doteq \int_0^{t^*} c(t)\,dt \tag{2}$$

is often used. Equation (2) is an equality only in the limit $t^* \to \infty$. One question to be considered here is the significance of the error caused by having finite t^*.

From the estimated $f^*(t)$ the estimated (cumulative) residence time distribution $F^*(t)$ is obtained from

$$F^*(t) = \int_0^{t^*} f^*(t)\,dt \tag{3}$$

For a linear system this is equivalent to the response to a saturation step, while $(1 - F^*(t))$ would be the response to an elution step. The value of t^* may differ in the three techniques (pulse, saturation step, and elution step), depending on analytical accuracy. This is usually inherently greatest in elution step tests, as the concentration of a tail need only be distinguished from zero. A pulse test attenuates rapidly unless the input pulse is very great, and a saturation step approaches a final, relatively large concentration.

The mean residence time in a system may be estimated from

$$\bar{t}^* = \int_0^{t^*} t f^*(t)\,dt = \int_0^1 t\,dF^* \tag{4}$$

and the second moment, from which the variance is obtainable, from

$$\overline{t^2}^* = \int_0^{t^*} t^2 f^*(t)\,dt = \int_0^1 t^2\,dF^* \tag{5}$$

The limit $t^* \to \infty$ gives, in all of the above cases, the fundamental relations between true values of $f(t)$, $F(t)$, $\bar{t}$, and $\overline{t^2}$. They are written in terms of t^* to emphasize the nature of the approximations we are forced to use.

A series of measurements were reported by Lapidus (*2*) and Schiesser and Lapidus (*4*) in which step and pulse tests through beds packed with both nonporous and porous spheres were used to obtain the holdup in the bed during trickle phase operation. They found agreement between pulse and step tests when determining

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